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Fitting:

A Subroutine to Fit Four-Moment Probability Distributions to Data

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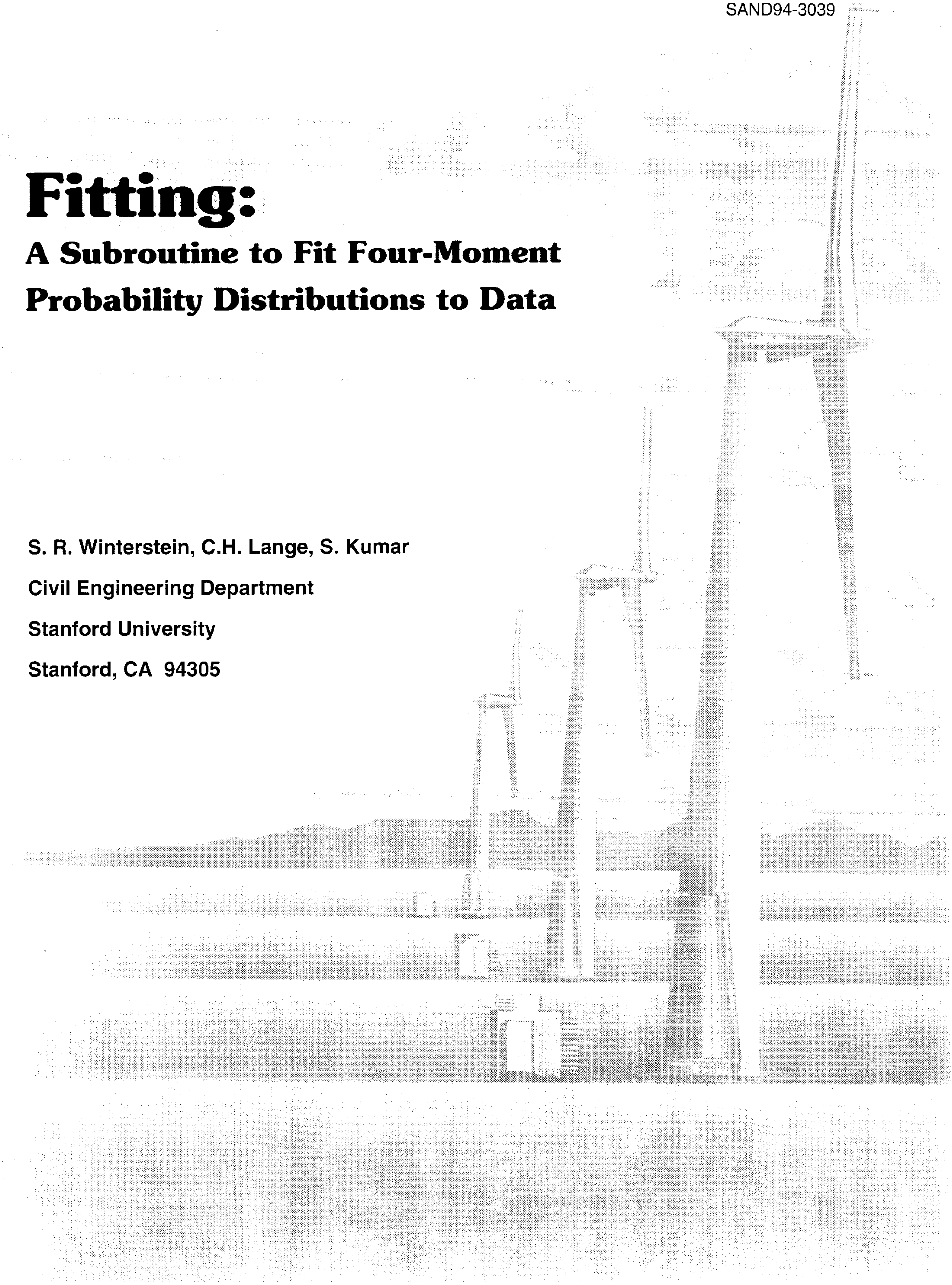

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FITTING: A SUBROUTINE TO FIT FOUR-MOMENT PROBABILITY DISTRIBUTIONS TO DATA

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Abstract

`fitting` is a Fortran subroutine that constructs a smooth, generalized four-parameter probability distribution model. It is fit to the first four statistical moments of the random variable X (i.e., average values of X, X^2, X^3, and X^4) which can be calculated from data using the associated subroutine `calmom`. The generalized model is produced from a cubic distortion of the parent model, calibrated to match the first four moments of the data. This four-moment matching is intended to provide models that are more faithful to the data in the upper tail of the distribution. Examples are shown for two specific cases.

Acknowledgement

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Contents

Executive Summary

`fitting` is a Fortran subroutine that constructs a smooth, generalized four-parameter probability distribution model. It is fit to the first four statistical moments of the random variable X (i.e., average values of X, X^2, X^3, and X^4) which can be calculated from data using the associated subroutine `calmom`.

This distribution is said to be "generalized" in that it generalizes three conventional, standard two-parameter "parent" distribution models. The user may select here between Gaussian, Gumbel, or Weibull parent models. The generalized model is produced from a cubic distortion of the parent model, calibrated to match the first four moments of the data. This four-moment matching is intended to provide models that are more faithful to the data in the upper tail of the distribution.

Examples are shown here for two specific cases: modelling rainflow-counted load ranges and extreme wave heights, based respectively on Weibull and Gumbel parent models. To use `fitting` to fit a distribution to data, a separate subroutine, `calmom`, is included to determine the first four statistical moments of the input data set. Because these moments are required input to `fitting`, the routines `calmom` and `fitting` together serve as a general distribution fitting algorithm. A sample driver program is included to illustrate the usage and interpretation of `fitting` and `calmom` for the two examples.

Chapter 1

The `fitting` Subroutine

1.1 What `fitting` Does

`fitting` is a Fortran subroutine that constructs a smooth, generalized four-parameter probability distribution model. The first four statistical moments of the random variable X (i.e., average values of X, X^2, X^3, and X^4) are used by the subroutine to establish the generalized distribution. These moments can be based on theory; however, they are almost always derived from data. A separate subroutine `calmom` is provided to compute the required moments for an arbitrary data set.

This distribution is said to be "generalized" in that it generalizes three conventional, standard two-parameter "parent" distribution models. The user may select here between Gaussian, Gumbel, or Weibull parent models. The generalized model is then produced from a cubic distortion of the parent model, calibrated to match the first four moments of the data. (Depending on the numerical values of the moments, an inverse cubic distortion may also be used.) This four-moment matching is intended to provide models that are more faithful to the extreme values of the data, commonly referred to as the upper tail region.

By invoking various parent models, the user is able to reflect reasonable "prior" probability distribution choices based on the context at hand. For example, values from a random process may be assigned Gaussian distribution if sampled at an arbitrary time, Weibull distribution if sampled at an arbitrary peak, or Gumbel distribution if sampled at a global peak in a fixed duration (Benjamin and Cornell, 1970). These three distributions are included here as possible parent distribution choices.

Examples are shown here for two specific cases: modelling rainflow-counted load ranges and extreme wave heights, based respectively on Weibull and Gumbel parent models. Notably, we find that over a range of practical values, these applications are controlled by the four moment values and are relatively insensitive to the underlying parent distribution choice. Because this conclusion may change with the application,

Values x_i [kip-ft]:	Probabilities $p_i = F(x_i)$:
9.250	0.004
9.500	0.058
9.750	0.146
10.000	0.251
10.250	0.360
10.500	0.465
10.750	0.561
11.000	0.645
12.000	0.862
13.000	0.949
14.000	0.981
15.000	0.993

Table 1.1: Predicted probabilities, p_i, of not exceeding various x levels.

`fitting` allows the user to implement various parent distributions and assess the sensitivity to this choice.

1.1.1 Overview of Capabilities

The subroutine `fitting` has two options. In the first option, the user can provide arbitrary values $x_1, ..., x_N$ of the physical variable, and the routine returns corresponding probability values, p_i, that the random variable is *less than* the value x_i. Formally, p_i is known as $F(x_i)$, the *cumulative distribution function (CDF)* of X. The second option works in the opposite direction: the user provides specified cumulative probability values p_i, and the routine returns levels x_i of the physical variable. In this case, the output levels x_i are known as specific *fractiles* of the probability distribution. Note that both options require the first four statistical moments of X to be input.

As a simple example, consider the edgewise bending moment range X on a wind turbine blade (Coleman, 1989). Figure 1.1 shows the cycle counts (rainflow counted) for a 71 minute time history of observed edgewise moments. The clustering of counts in the moment ranges around 9-15 and 0-5 kip-ft is attributed to the dominant gravity induced loading combined with the turbulent effects of the wind respectively. From the viewpoint of fatigue damage, ranges less than 9 kip-ft contribute less than 5% to the total damage and are considered insignificant. Table 1.1 shows the cumulative distribution, $F(x)$, of applied loads above this level as predicted by `fitting`.

For example, Table 1.1 shows that among all loads above `xmin`=9 [kip-ft], the load level 9.5 [kip-ft] is not exceeded 5.8% of the time—and hence exceeded the remaining

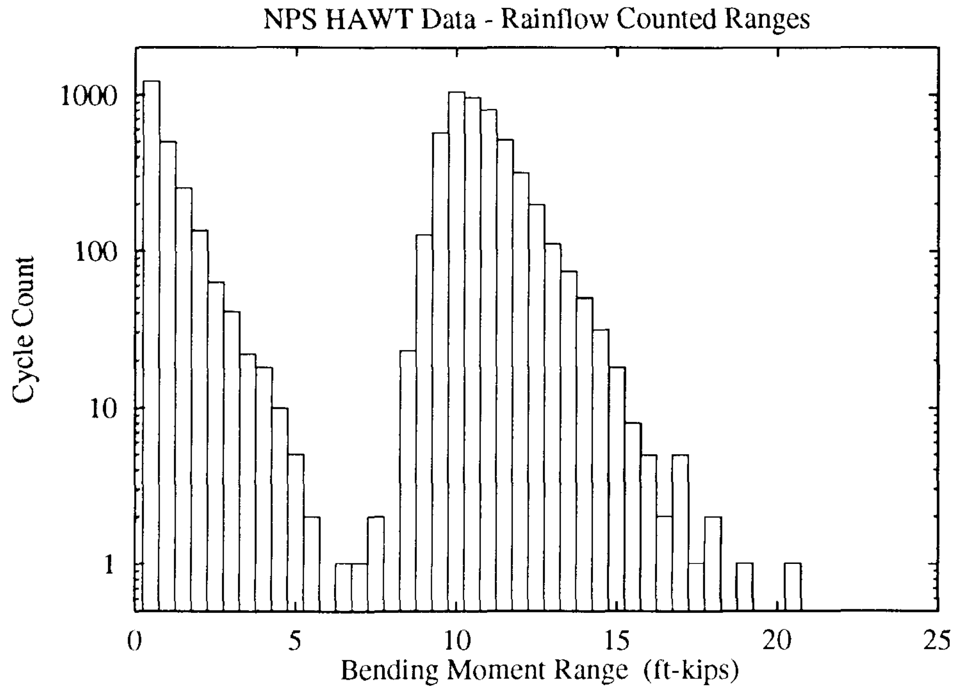

Figure 1.1: Histogram of Edgewise Bending Moment Time Series Data

94.2% of the time. Similarly, this table shows that a typical, central load value is around 10.5 [kip-ft]. Strictly, this value is not exceeded 46.5% of the time; i.e., the *median* value of the load X—which has 50% chance of being exceeded–is between 10.5 and 10.75 [kip-ft]. We show in Section 2.2 how these values are estimated from the `fitting` routines. We also show how, if we invoke the second option of `fitting`, we can directly estimate the median level (for which $F(x)=p_i$ is specified to be 0.5) to be 10.59 [kip-ft]. Notice also from Table 1.1 that probabilities for values that exceed the range of observed values can be requested.

In general, one may consider three distinct ways to use `fitting`:

- One option of the subroutine `fitting` takes input values x_i and estimates the cumulative probability p_i of falling below each x_i. For example, data in the first column of Table 1.1) are input and the second column values are output.
- With this same option, the differences $p_i - p_{i-1}$ between these cumulative probabilities can be used to give estimates of a theoretical histogram (i.e., probability content in various discretized "cells" or "bins.") For example, Table 1.1 can be used to directly produce a histogram, with probability .032 assigned to the interval (9.25, 9.50), probability .126-.032=.094 to the interval (9.50, 9.75), and so forth.
- The other option of the subroutine `fitting` takes input values p_i and returns corresponding values x_i. For example, data in the second column of Table 1.1

are input and the first column values are output. This is useful, for example, if the user can more easily specify interesting values of p_i rather than x_i values a priori.

1.1.2 Subroutine `calmom`

`fitting` requires the first four statistical moments of the random variable, X, as input. These moments can be based on either theoretical considerations or derived from a particular set of data. To use `fitting` to fit a distribution to data, a separate subroutine, `calmom`, is included to accurately estimate the first four statistical moments of the input data set. Input and output for the `calmom` subroutine are described in Section 1.3.

Because these moments are required input to `fitting`, the routines `calmom` and `fitting` together serve as a general distribution fitting algorithm. A sample driver program is included to illustrate the usage and interpretation of `fitting` and `calmom` for two example problems given in Chapters 2 and 3.

1.1.3 How to Read or Not Read This Manual

We recognize that there are two distinct types of computer users: those who read manuals thoroughly and those who go to great lengths to avoid doing so. For this latter group, who prefer to learn by example, we have included a driver program with detailed comments, and the sample input and output used to generate Table 1.1. Those users may wish to proceed to the driver source code listing, also given in Appendix A. Additional description of the driver and this example, based on a generalized Weibull model, is given in Chapter 2. Chapter 3 describes an alternate application to extreme wave height levels, using a generalized Gumbel model.

Those who prefer a more precise overview of `fitting` are referred to the remainder of Chapter 1. Section 1.2 describes its input and output arguments and calling syntax, while Section 1.3 discusses the usage and arguments of the subroutine `calmom` which computes statistical moments for a given data set.

Finally, Chapter 4 brings together a number of more detailed technical issues concerning `fitting`. These range from underlying motivation (Section 4.1) to the basic methodology underlying `fitting` (Section 4.2) and `calmom` (Section 4.3). Section 4.4 includes various additional practical notes on their usage, limitations and potential error conditions.

1.2 fitting Input and Output

The user can call `fitting` with the following command:

```
call fitting(itype,xmom,ndata,xmin,x,cdf,nx,pmom,iflag,ioout,etol)
```

Each component of the `fitting` argument list is defined below. Output quantities include the array `pmom` and, depending on the value of `iflag`, either `x` or `cdf`. All other quantities are input.

- `itype`: index used to define the parent distribution used by `fitting`

  `itype` = 1: Gaussian distribution

  `itype` = 2: Gumbel distribution

  `itype` = 3: Weibull distribution

- `xmom(1)`[1]: mean, $\mu_x = E[X] = \int_{all\ x} x f(x) dx$; $f(x)$=PDF of X
- `xmom(2)`[1]: standard deviation, $\sigma_x = \{E[(x-\mu_x)^2]\}^{1/2} = \{\int_{all\ x}(x-\mu_x)^2 f(x)dx\}^{1/2}$
- `xmom(3)`[1]: skewness, $\alpha_3 = E[(\frac{x-\mu_x}{\sigma_x})^3] = \int_{all\ x}(\frac{x-\mu_x}{\sigma_x})^3 f(x)dx$
- `xmom(4)`[1]: kurtosis, $\alpha_4 = E[(\frac{x-\mu_x}{\sigma_x})^4] = \int_{all\ x}(\frac{x-\mu_x}{\sigma_x})^4 f(x)dx$
- `ndata`[1]: Number of sample data used to estimate moments in `xmom`. If `ndata`$<$ 100, `fitting` checks by simulation that these moments do not have excessive bias (Section 4.3). The user can avoid this simulation check by setting `ndata` $\geq$ 100 on input.
- `xmin`: Optional shift parameter to be applied in the Weibull case (`itype`=3). Note that the standard Weibull model produces values for $X \geq 0$, while the Gaussian and Gumbel models are unbounded. If the user inputs a nonzero value of `xmin`, a shifted Weibull model (standard Weibull model of $X-$ `xmin`) is then used as a parent distribution. Accordingly, in this case `xmom(1)` ... `xmom(4)` should contain moments of the shifted variable $X-$ `xmin`. (This is precisely what is returned by the routine `calmom` when `xmin` is nonzero.) The data shown in figure 1.1 is a good example of using this variable (`xmin` $\approx$ 9.0) when only the upper portion of the data is important. Note finally that `fitting` ignores the value of `xmin` if the Gaussian or Gumbel distribution is selected.
- `x`: array containing values of the physical variable.

[1]Section 1.3 explains these moments further and subroutine `calmom` used to compute them

- `cdf`: array containing the cumulative (non-exceedance) probabilities corresponding to each x_i in the `x` array above.

- `nx`: number of `x` or `cdf` values requested.

- `pmom`: array of the absolute moments of the fitted distribution:

  `pmom(n)`: the n-th absolute moment, $E[X^n] = \int_{all\ x} x^n f(x)dx$

  The first four moments will be consistent with the input moments given in `xmom`, within the error tolerance described below. Higher moments may be of interest in other applications; e.g., fatigue damage of various materials. Here n=10 moments are output, using the probability density function $f(x)$ estimated by the Generalized distribution model.

- `iflag`: index used to define the type of calculation to be performed by `fitting`

  `iflag` = 0: returns output estimates of `x` for each of the cumulative probabilities input in the array `cdf`.

  `iflag` = 1: returns output estimates of probabilities `cdf` for each of the physical values input in the array `x`.

- `ioout`: logical unit number for writing error messages. The calling program should make a file available for error messages by opening a file with `ioout` as its logical unit number. (The sample driver illustrates this in Chapter 2.)

- `etol`: the error tolerance in matching higher moments. This is defined formally in Eq. 4.2. In general, there is a tradeoff between moment accuracy and computation time. Based on experience with various tolerances, we use the value `etol`=.01 in our examples. This may be changed by the user.

If the theoretical moments `xmom(i)` are known, the `fitting` routine can be applied directly. In practice, it is often necessary to estimate these statistical moments from a set of data. A separate subroutine, `calmom`, is supplied here to compute the required moments from data; i.e., to act as a pre-processor for `fitting` routine. The procedure used to compute these moments is discussed in section 4.3, and its use is demonstrated in Chapter 2.

The Role of the Lower Threshold `xmin`. In most applications of the Weibull model we seek to model all possible values of a positive quantity (e.g., stress range, number of cycles to failure, etc.). In certain applications, however, the user may wish to impose a non-zero lower-bound `xmin`. This is useful, for example, if we wish to exclude lower values as non-physical, or due to a fundamentally different probability distribution. We have found this useful, for example, in modelling some edgewise bending loads on a turbine blade. In this case, we seek to exclude small, non-damaging loads produced by a different mechanism: low amplitude (turbulence induced) cycles superimposed on marked gravity-driven bending moment ranges. This case is illustrated further in the example of Chapter 2.

1.3 calmom Input and Output

The subroutine calmom estimates the first four statistical moments $xmom(i)$, i=1...4, from an input data set. It can thus serve as a preprocessor to fitting.

The calmom argument list is:

```
subroutine calmom(xmom,data,ndata,nrmax,xmin,itype)
```

The input to calmom are the following:

- data: array containing the data for which the moments are to be calculated.
- ndata: number of data points in array data.
- nrmax: dimension size of array data (should be consistent with that used in calling program).
- xmin: threshold value of the physical variable, as used in fitting (see description of xmin in section 1.2 and the example problem in Chapter 2).
- itype: index used to define the parent distribution used by fitting

  itype = 1: Gaussian distribution

  itype = 2: Gumbel distribution

  itype = 3: Weibull distribution

The sample data input via the array data, can be arranged in any order and does not need to be sorted in increasing magnitude as shown in the example input of Table 2.1. Also, calmom screens the array data removing any values that are below the threshold xmin.

On output the array xmom contains the sample moments of the data, as defined in section 1.2. These can then be used directly as input to the routine fitting. The theoretical background for calmom is described in Section 4.3.

1.4 The Driver Program

A single driver program is used to demonstrate the use of subroutines fitting and calmom for two examples.

In general, the source code is distributed in three separate files:

calmom.for: The `calmom` subroutine to estimate moments from an input data set.

fitting.for: The `fitting` subroutine, to use these moments to estimate the entire distribution function of X.

driver.for: A separate driver program that calls these routines (listing in Appendix A).

This driver program is included to help speed the reader's understanding and implementation of `fitting`. The example shown here can thus be run without creating any additional source code. One needs merely to compile and link the three source codes listed above, and execute with the input files provided.

Of course, prospective users are encouraged to modify the driver program according to their needs. Toward that end, it is hoped that `driver.for` can provide a useful template. For those users who prefer to learn by example, we recommend reading the source code of `driver.for` as a useful starting point.

Analysis Steps. As implemented in `driver.for`, the analysis proceeds in the following steps:

1. Read control data: `itype`, `xmin`, `iflag`, and the array `cdf` or `x` used as input to `fitting`.
2. Read input data: the array `data` used as input to `calmom`, which calculates the necessary moment input for `fitting`.
3. Call `calmom` to estimate moments.
4. Call `fitting` to estimate the corresponding full distribution function.
5. Write results.

These steps are clearly delineated in comments contained within the source code of `driver`.

File Architecture. In the current coding of the driver (Appendix A), two input files are expected:

driver.in: Input file containing control data read in step 1 above.

driver.dat: Input file containing physical data read in step 2 above.

Together with the three source code files, we are distributing input files for two examples: (1) `weibull.in`, `weibull.dat`; and (2) `gumbel.in`, `gumbel.dat`. The user should note that to implement one of these examples, the input files `*.in` and `*.dat` (*='weibull' or 'gumbel') should be copied to `driver.in` and `driver.dat` before executing. The corresponding output file, `driver.out`, should then agree with the file `*.out` that has been distributed.

The examples described in Chapters 2 and 3 provide tables that identify more clearly the contents of the files `driver.in` and `driver.dat`.

Chapter 2

Fatigue Load Modelling: A Generalized Weibull Example

This chapter describes the first example, which relates to fatigue load modelling. The next chapter describes an alternate application to extreme wave height modelling.

2.1 Wind Turbine Loads Example

This example concerns the edgewise bending moments shown in figure 1.1 of Chapter 1 (Coleman, 1989). We consider here 8913 values of X=bending moment range [kip-ft], as found by rainflow counting (Fuchs and Stephens, 1980). The data are stored in the file `weibull.dat`. Table 2.1 gives a partial listing of these values. For the sake of clarity they are input in increasing order; however, this is not required by the program.

A separate analysis of these data (Winterstein and Lange, 1994) has shown that bending moment ranges below 9 kip-ft do not contribute to the total fatigue damage given by this data set. Since the application for the load model is a fatigue analysis of the HAWT blade, we choose to fit the model above a lower threshold `xmin`=9 [kip-ft]. Note that only 4819 ranges are above this threshold.

This threshold is set in the first line of the input file `weibull.in`. Table 2.2 lists this file. The first line also contains the values `iflag`=1 and `itype`=3. The value `iflag`=1 indicates that values of `x` are to follow on the subsequent lines in the file, and the program is to calculate corresponding `cdf` values. The value `itype`=3 tells the `fitting` routine to select Weibull as the parent distribution. The remaining lines list the actual `x` values requested, which are the same as in column 1 of Table 1.1.

Output. The driver produces a single output file, `driver.out`, which we have stored here as `weibull.out`. Table 2.3 lists `weibull.out` for our example. The

Line number i:	Data x_i [kip-ft]:
1	0.0190
2	0.0190
3	0.0190
.	.
.	.
.	.
4092	8.9810
4093	8.9820
4094	8.9820
4095	9.0010
4096	9.0010
4097	9.0010
.	.
.	.
.	.
8911	18.0420
8912	18.9290
8913	20.4990

Table 2.1: Abridged listing of edgewise moment ranges [kip-ft] from `weibull.dat`. File contains column 2 data only; line numbers are inserted here for clarity.

```
 9.00   1 3 ; XMIN=LOWER THRESHOLD; IFLAG=1 TO GET CDF FOR GIVEN X; ITYPE=3 TO FIT GENERALIZED WEIBULL
 9.25
 9.50
 9.75
10.00
10.25
10.50
10.75
11.00
12.00
13.00
14.00
15.00
```

Table 2.2: Listing of input file, `weibull.in`, with control data for driver program.

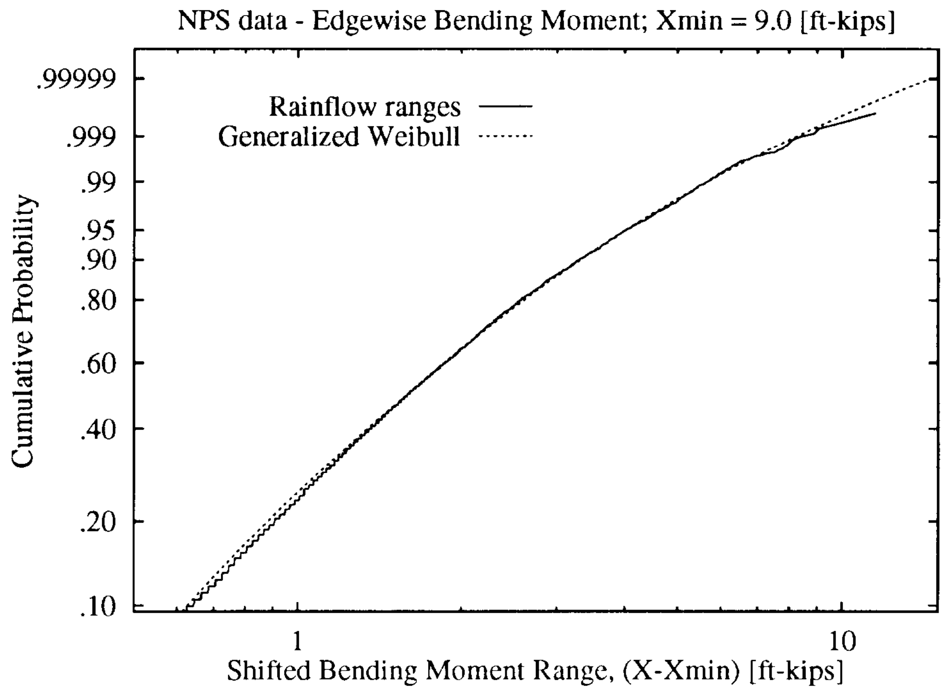

Figure 2.1: Generalized Weibull Distribution for Fatigue Loads–4819 Data.

fractile results reported by `fitting` are precisely those given in Chapter 1 (Table 1.1). Note also that the output confirms that 4819 data points have been found above the input lower threshold of `x`=9. It also reports the first four moments from these data, as estimated by `calmom`, that are used as input to `fitting`. Finally, the model predicts the first 10 absolute moments, $E[X^n]$. Note that these are consistent with the first four moments found for the data. For example, $E[X^1]$=1.831, the mean value μ_x, while $E[X^2]=\sigma_X^2 + \mu_X^2$, or $1.154^2 + 1.831^2$=4.685. Similarly, the predicted third and fourth moments can be shown to be consistent with those estimated from the data. The main virtue of the routine, of course, is that it seeks to predict still higher moments more accurately—through introduction of a smooth distribution model—than would be possible from the data alone.

Figure 2.1 compares the fitted distribution function $F(x)$ with the data. (These results have been obtained by running the `fitting` routine over a larger range of x values than shown in the example.) Results are shown on "Weibull probability scale," on which the parent Weibull model appears as a straight line. It appears that the generalized Weibull model reflects the curvature of the data shown on this scale, particularly in the upper tail of interest (which is most heavily weighted by the third and fourth moments).

```
   Lower Threshold Value:     9.000
Number of Data Processed:  4819

      **  MOMENT RESULTS  **

                   Mean:      1.831
     Standard Deviation:      1.154
               Skewness:      1.552
               Kurtosis:      7.449

   ** FRACTILE RESULTS (FITTING) **

                   X:         CDF:

                  9.250       0.004
                  9.500       0.058
                  9.750       0.146
                 10.000       0.251
                 10.250       0.360
                 10.500       0.465
                 10.750       0.561
                 11.000       0.645
                 12.000       0.862
                 13.000       0.949
                 14.000       0.981
                 15.000       0.993

  ** PREDICTED MOMENTS (FITTING) **

                   N:      E[X**N]:

                  1.000 0.183E+01
                  2.000 0.469E+01
                  3.000 0.159E+02
                  4.000 0.688E+02
                  5.000 0.377E+03
                  6.000 0.259E+04
                  7.000 0.221E+05
                  8.000 0.234E+06
                  9.000 0.302E+07
                 10.000 0.472E+08
```

Table 2.3: Listing of output file, `weibull.out`, produced by driver program.

```
9.00   0 3 ; XMIN=LOWER THRESHOLD; IFLAG=0 TO GET X FOR GIVEN CDF; ITYPE=3 TO FIT GENERALIZED WEIBULL
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
0.99
0.999
```

Table 2.4: Listing of input file for iflag=0 option.

2.2 Alternate Usage of fitting

Finally, we illustrate the iflag=0 option of fitting. For example, if the **weibull.in** content is modified as shown in Table 2.4, Table 2.5 shows the corresponding output. The data file **weibull.dat** remains the same, and hence all moment results are unchanged. The only difference is that in this case, the distribution fractiles **x** have been evaluated at the requested probability levels given in the input file **driver.in** (Table 2.4). For example, as noted in Chapter 1, the median value of X (with 50% chance of being exceeded) is found to be 10.589. The values of bending moment that are not exceeded 99% and 99.9% of the time were also determined.

```
   Lower Threshold Value:     9.000
Number of Data Processed:   4819

      **  MOMENT RESULTS  **

                   Mean:      1.831
     Standard Deviation:      1.154
               Skewness:      1.552
               Kurtosis:      7.449

   ** FRACTILE RESULTS (FITTING) **

                     X:          CDF:

                 9.627       0.100
                 9.882       0.200
                10.114       0.300
                10.345       0.400
                10.589       0.500
                10.861       0.600
                11.188       0.700
                11.620       0.800
                12.324       0.900
                14.664       0.990
                17.255       0.999

  ** PREDICTED MOMENTS (FITTING) **

                     N:      E[X**N]:

                 1.000 0.183E+01
                 2.000 0.469E+01
                 3.000 0.159E+02
                 4.000 0.688E+02
                 5.000 0.377E+03
                 6.000 0.259E+04
                 7.000 0.221E+05
                 8.000 0.234E+06
                 9.000 0.302E+07
                10.000 0.472E+08
```

Table 2.5: Listing of output file, generated from `iflag=0` input file given in Table 2.4.

Chapter 3

Extreme Values: A Generalized Gumbel Example

This chapter illustrates the use of the `fitting` routine to fit a generalized Gumbel distribution to extreme values. The driver program used for this demonstration is described in Chapter 1. This driver program reads the relevant input data for this example and passes them to the `calmom` and `fitting` routines to construct the generalized Gumbel distribution.

This example concerns the significant wave height H_s in a Southern North Sea location, for which 19 years of hindcast data are available (Danish Hydraulic Institute, 1989). For each of these 19 years, a single storm event has been identified with maximum significant wave height H_s (i.e. the extreme values). This value ranges from $H_s = 6.92$m (1972/1973) to 9.66m (1981/1982). A sorted list of all 19 values is reported in Table 3.1.

This chapter has two sections. The first section deals with the generalized Gumbel model for the significant wave height data. The second section compares the three different generalized distribution models for the same data set.

Finally, it should be noted that a generalized Gumbel model has previously been fit to this data set (Winterstein and Haver, 1991). The results shown here are an improvement in two senses: (1) `fitting` permits greater accuracy to be achieved in matching moments; and (2) `fitting` includes an inverse cubic transformation, which is particularly important in reflecting the narrower-than-Gumbel tails similar to the data in Table 3.1.

Line number i:	Data H_{si} [m]:
1	9.66
2	9.44
3	9.18
4	9.17
5	8.85
6	8.79
7	8.60
8	8.58
9	8.54
10	8.49
11	8.09
12	8.08
13	8.06
14	7.47
15	7.42
16	7.41
17	7.31
18	7.16
19	6.92

Table 3.1: Listing of annual significant wave height [m] from `gumbel.dat`. File contains column 2 data only; line numbers are inserted here for clarity.

3.1 Generalized Gumbel Results

The annual significant wave height data consists of 19 values listed in Table 3.1. The data are stored in the file `gumbel.dat`. For the sake of clarity they are input in decreasing order; however, this is not required by the program.

The control data are stored in `gumbel.in`. Table 3.2 lists this file. The first line of this file contains three values. The first value is `xmin`=0.0, which sets the lower threshold value. Note, however, that this is not used in this case of Gumbel distribution since there is no cutoff value. This threshold value is used when generalized Weibull distribution is fit to the data (see Chapter 2). The second argument, `iflag`=1, indicates that `x` values are to follow in the file and the program will calculate corresponding `cdf` values. The third argument, `itype`=2, indicates that a generalized Gumbel distribution is to be fit to the data in file `gumbel.dat`. The remaining lines list the actual `x` values requested.

Output. Table 3.3 lists the corresponding output file `gumbel.out` for this example. It also reports the first four moments from these data, as estimated by `calmom`,

```
 0.00  1 2 ; XMIN=LOWER THRESHOLD; IFLAG=1 TO GET CDF FOR GIVEN X; ITYPE=2 TO FIT GENERALIZED GUMBEL
10.00
 9.66
 9.44
 9.18
 9.17
 8.85
 8.79
 8.60
 8.58
 8.54
 8.49
 8.09
 8.08
 8.06
```

Table 3.2: Listing of input file, `gumbel.in`, with control data for driver program.

which are used as input to `fitting`. Finally, the model predicts the first 10 absolute moments, $E[X^n]$. Note that these are consistent with the first four moments found for the data. For example, $E[X^1] = 8.275$, the mean value μ_x, while $E[X^2] = \sigma_X^2 + \mu_X^2$, or $.819^2 + 8.275^2$=69.1. Similarly, the predicted third and fourth moments can be shown consistent with those estimated from the data.

As discussed in Chapter 2, note that to use the input files `gumbel.in` and `gumbel.dat` with the driver program they must be copied to the files `driver.in` and `driver.dat` respectively. The output, written to `driver.out`, should then be comparable to `gumbel.out`.

In order to generate a smooth plot of the generalized Gumbel distribution, an input file similar to `driver.in` with a greater number of input values to compute corresponding CDF values was used. This distribution is plotted in Figure 3.1 along with the observed data values. It appears to capture fairly well the systematic curvature of the data on the Gumbel probability scale used.

3.2 Comparison of Three Generalized Distributions for Wave Height

Because we deal here with annual extreme values, the Gumbel distribution is the natural choice of parent distribution. We may ask, however, what effect is achieved if

```
Number of Data Processed:  19

      **  MOMENT RESULTS  **

                   Mean:      8.275
     Standard Deviation:      0.819
               Skewness:     -0.053
               Kurtosis:      1.905

   ** FRACTILE RESULTS (FITTING) **

                   X:          CDF:

                10.000       0.998
                 9.660       0.978
                 9.440       0.937
                 9.180       0.846
                 9.170       0.842
                 8.850       0.690
                 8.790       0.662
                 8.600       0.580
                 8.580       0.572
                 8.540       0.557
                 8.490       0.539
                 8.090       0.427
                 8.080       0.424
                 8.060       0.419

  ** PREDICTED MOMENTS (FITTING) **

                   N:      E[X**N]:

                 1.000 0.827E+01
                 2.000 0.691E+02
                 3.000 0.583E+03
                 4.000 0.496E+04
                 5.000 0.426E+05
                 6.000 0.369E+06
                 7.000 0.322E+07
                 8.000 0.282E+08
                 9.000 0.250E+09
                10.000 0.222E+10
```

Table 3.3: Listing of output file, `gumbel.out`, produced by driver program.

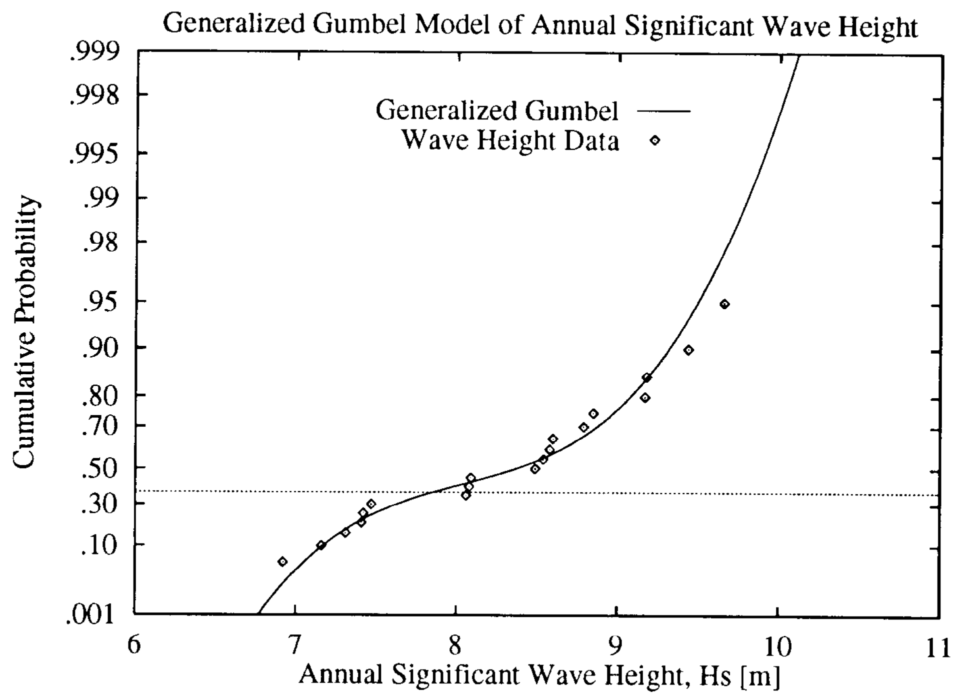

Figure 3.1: Generalized Gumbel Distribution for Annual Extreme Wave Height–19 Data.

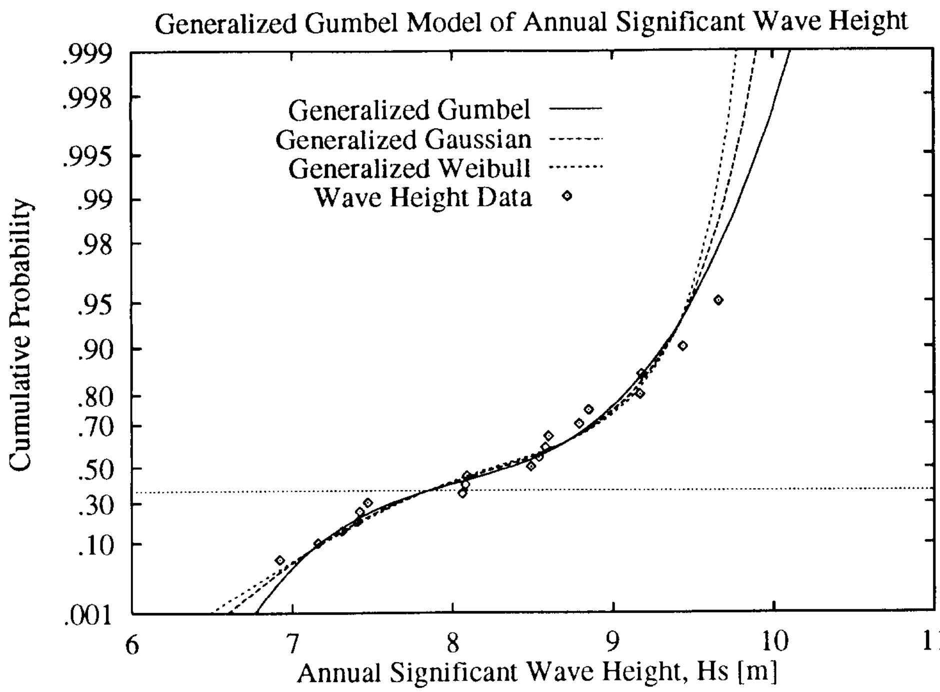

Figure 3.2: Comparison of Generalized Gaussian, Gumbel, and Weibull Distributions for Annual Extreme Wave Height.

a different choice of parent distribution is selected. This is investigated in this section.

We again use the same data set as listed in Table 3.1. Thus the input data file is same as `driver.dat` of Section 3.1. However, the control input file `driver.in` is varied so that `itype` is either 1, 2, or 3. These three different values of `itype` give three generalized distributions: generalized Gaussian (`itype` = 1), generalized Gumbel (`itype` = 2), and generalized Weibull (`itype` = 3).

The three distribution are shown in Figure 3.2. The figure shows wave height results up to the 1000-year fractile, i.e. for which p=.999 and hence $-\ln(-\ln(p))$=6.9. The pattern of variation follows that of the underlying parent distributions: the Weibull has the narrowest upper tail and hence predicts the lowest extreme values, while the Gumbel predicts the largest. Most notably, however, all three parent distributions predict quite similar wave heights over this domain of interest.

This suggests that knowledge of four moments is sufficient to control the tail behavior of interest. This apparent robustness of the four-moment description is encouraging, particularly in cases where the optimal parent distribution is not obvious. Of course this conclusion may be problem-dependent; the user is encouraged to vary the choice of `itype` for the problem at hand.

Chapter 4

Technical Background and Additional Details

4.1 Motivation

The `fitting` routine has been developed to modify standard, commonly used distributions to better match observed tail behavior. In particular, cubic distortions of these standard "parent" distributions are sought to match the first four moments of the data. We may then ask why precisely *four* moments are used to fit the probability distribution of X—and not two, three, five, ten, etc. Conventional models are of lower order, requiring only one or two moments. The problem is that a number of plausible models, with very different tail behavior and hence fatigue reliability, can be fit to the same first two moments. This scatter in reliability estimates is said to be produced by *model uncertainty.* This is prevalent in low-order, one- or two-moment models. (Note that many common fatigue load models include only one parameter; e.g., the Rayleigh and exponential models.)

To avoid this model uncertainty, which is difficult to quantify, one is led to try to preserve higher moments as well. This will help to discriminate between various models, and hence reduce model uncertainty. The benefit does not come without cost, however: higher moments are more sensitive to rare extreme outcomes, and hence are more difficult to estimate from a limited data set. This is known as *statistical uncertainty,* which reflects the limitations of our data set.

Thus, our search for an "optimal" model reflects an attempt at balance between model and statistical uncertainties. Practical experience (e.g., Winterstein, 1988) suggests that four moments are often sufficient to define upper distribution tails over the range of interest. This experience motivates the generalized models developed here. It is again supported by the results of Section 3.2, in which extreme wave heights are insensitive to the choice of parent distribution, once four moments have

4.2 Underlying Methodology: `fitting`

The **`fitting`** routine begins with a theoretical, 2-parameter "parent" distribution. In the current implementation, the user may choose Gaussian, Gumbel, or Weibull parent distributions. Denoting this parent variable as U, **`fitting`** then models the physical random variable X through a cubic transformation of U:

$$X = c_0 + c_1 U + c_2 U^2 + c_3 U^3 \tag{4.1}$$

The optimizer adjusts the coefficients c_n through an iterative scheme until the difference between the requested skewness, α_3, and kurtosis, α_4, (see **`xmom(3)`** and **`xmom(4)`**, section 1.2) and those of the generalized model in Eq. 4.1 (α_{3X} and α_{4X}) are minimized.

The optimizer also restricts the coefficients so that Eq. 4.1 remains monotonically increasing, producing a well-behaved model that only mildly modifies the underlying parent distribution. This leads, for example, to requiring $c_3 \geq 0$ so that X in Eq. 4.1 continues to grow as U becomes large. This in turn makes it difficult to model cases with tails that are narrower than those of the parent distribution. In particular, it is difficult to use Eq. 4.1, with positive c_3, to model situations in which the desired kurtosis, α_4, is less than that of the parent variable, α_{4U}. In this case **`fitting`** inverts the model, seeking to fit a model analogous to Eq. 4.1 in which the roles of X and U are interchanged. (In this view, one seeks to expand the distribution tail of the *actual* variable X to produce a parent variable U, so that c_3 becomes positive.)

Note that this switching between two dual models, based on the size of α_4, occurs automatically within **`fitting`** and should be of no consequence to the user. Adding such a dual model, however, has has been found to greatly increase modelling flexibility for small kurtosis cases. These have been found to arise both in extreme and fatigue loading applications.

Finally, in whichever form the model is defined, the coefficients c_n are chosen to minimize the error ϵ, defined as

$$\epsilon = \sqrt{(\alpha_3 - \alpha_{3X})^2 + (\alpha_4 - \alpha_{4X})^2} \tag{4.2}$$

The speed of executing **`fitting`** is governed largely by the speed of this optimization; i.e., by the amount of effort (trial c_n values) needed to achieve an acceptably small tolerance, ϵ_{tol}. The driver program sets ϵ_{tol}=.01 for the examples shown. The user can vary this tolerance, with the resulting change in computation time to be expected.

4.3 Underlying Methodology: `calmom`

To motivate the need for this routine, we must consider a brief background in statistical moment estimation. If we seek to estimate the ordinary mean value $E[X]$=μ

from data $X_1 \dots X_n$, a natural estimate is the simple average value $\overline{X}=\sum_{i=1}^{n} X_i/n$. Similarly, the k-th order "ordinary" moment, $E[X^k]$, is naturally estimated by the corresponding average $\sum_{i=1}^{n} X_i^k/n$.

The difficulties arise when we instead seek, as in many applications, to estimate not ordinary but *central* moments; i.e., of the form $\mu_k=E[(X-\mu)^k]$ for k=2, 3, 4.... (Note that `fitting` input stops at k=4: $\sigma_x=\mu_2^{1/2}$, $\alpha_3=\mu_3/\mu_2^{1.5}$, and $\alpha_4=\mu_4/\mu_2^2$.)

The problem here lies in its circular aspect: we must first estimate the unknown first moment μ before seeking to estimate $\mu_k=E[(X-\mu)^k]$. And, if we use the same data set for both purposes, we typically find too-low estimates of μ_2, μ_3, μ_4, etc. because our μ value is artificially tuned to best match the mean of the observations. Those exposed to a standard statistics course will best recognize this phenomenon when estimating the variance μ_2: to inflate the sample variance to account for this bias, the sum of squared deviations is divided by $n-1$ rather than n.

While unbiased estimates of the higher moments μ_3, μ_4, ... are less familiar, they are available in the statistical literature (Fisher, 1928):

$$\mu = \overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i \tag{4.3}$$

$$\mu_2 = \frac{n}{n-1} m_2 \tag{4.4}$$

$$\mu_3 = \frac{n^2}{(n-1)(n-2)} m_3 \tag{4.5}$$

$$\mu_4 - 3\mu_2^2 = \frac{n^2}{(n-1)(n-2)(n-3)}[(n+1)m_4 - 3(n-1)m_2^2] \tag{4.6}$$

in terms of the sample central moment $m_k=\sum_{i=1}^{n}(X_i-\overline{X})^k/n$. Eq. 4.4 is the conventional result for the sample variance.

Remaining Bias. Finally, the routine calmom uses these results to estimate the quantities σ_x by $\mu_2^{0.5}$, α_3 by $\mu_3/(\mu_2^{1.5})$, and α_4 by $\mu_4/(\mu_2^2)$. Because these vary nonlinearly with μ_n, they may still contain some bias although the μ_n estimates do not.

For example, if we fit a Gumbel model to the 19 wave height data from Chapter 3, the true skewness and kurtosis values are 1.14 and 5.40. However, simulating 10000 data sets of size n=19 and running each through calmom, we find on average the skewness 0.79 and kurtosis 3.89 (Winterstein and Haver, 1991).

To address this problem, the `fitting` routine has an automatic check for remaining bias through simulation. This is why `ndata` is given as an input parameter. After `fitting` constructs a distribution with moments from the input array `xmom`, many similar data sets (of size `ndata`) are simulated from this distribution. If the moments

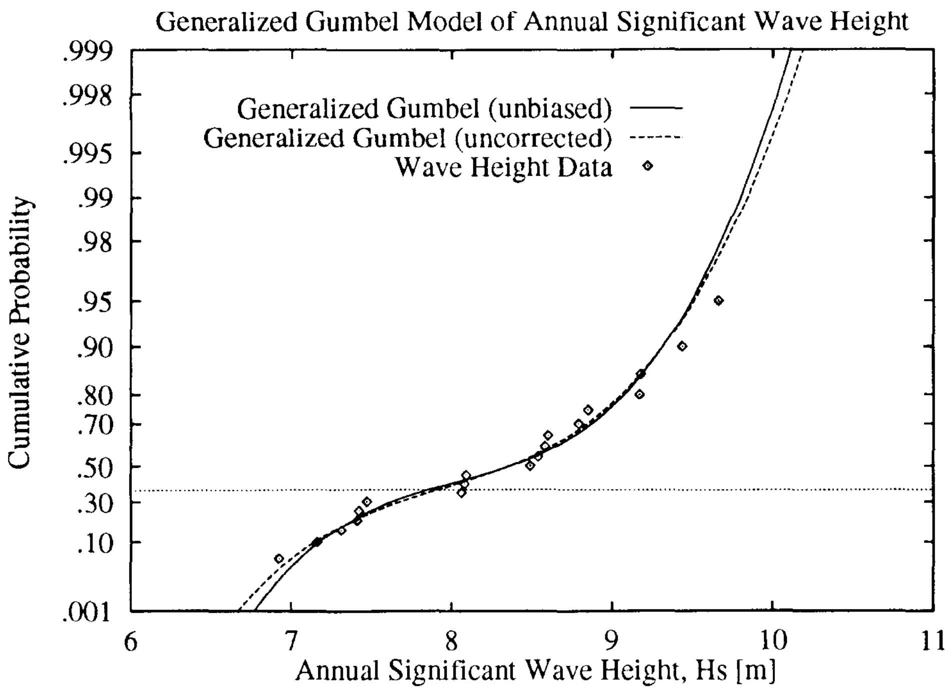

Figure 4.1: Effect of Ignoring Bias: Wave Height Example.

predicted from `calmom` differ appreciably on average from the input values, new theoretical estimates of the moments are constructed. This estimation-simulation loop is continued iteratively until satisfactory convergence is found.

Note that the `fitting` routine does not perform this simulation if its input parameter `ndata` ≥ 100. This value can be hard-wired if the user wishes to bypass this option. Figure 4.1 shows the effect of enabling this "unbiased" option (the default) and disabling it (using "raw" moments from `calmom` directly) for the generalized Gumbel model produced for the example given in Chapter 3. There is relatively little difference found in these cases. Larger effects may be found for cases of (1) fewer data and/or (2) distributions with broader tails.

4.4 Notes on Usage

The `fitting` routine has limiting conditions that users should note. When these conditions are encountered, appropriate error messages are written to the output file/device corresponding to the input logical unit number `ioout`. This section explains the meaning of these messages and discusses other details regarding `fitting` usage.

4.4.1 Errors in Matching Moments

Note that in most practical cases, the coefficients c_n in Eq. 4.1 can be chosen so that the error ϵ, as given in Eq. 4.2, falls within the user-defined tolerance limit ϵ_{tol}. In rare cases the minimized error exceeds ϵ_{tol}. In these cases `fitting` writes an error message indicating the magnitude of ϵ, the error norm of the skewness and kurtosis in Eq. 4.2.

4.4.2 Lower Tail Limiting Values

Lower tail limiting values are only a problem when the parent distribution is Weibull. In this case the variable U in Eq. 4.1 has Weibull distribution, and hence a minimum value of 0. Because Eq. 4.1 is monotonic, the corresponding smallest possible value of the physical variable X is c_0. This physical lower limit c_0 can be either greater or less than zero, since the optimized Weibull model in Eq. 4.1 will not in general have its x intercept at exactly zero.

This may lead to situations that seem anomalous. If the lower limit c_0 is negative, for example, `fitting` may estimate negative values of X for probabilities near zero. Conversely, if the lowest possible value c_0 is positive, an input X value below c_0 cannot occur and `fitting` will return a zero cumulative probability (CDF=0). When `xmin` is not zero the situation is entirely analogous: c_0 may be greater or less than `xmin`.

In practice we believe this to be a minor issue for the following reasons:

- The routine `fitting` is intended for applications where large X values (upper distribution tails) are crucial. This is the motivation for preserving higher moments. Its accuracy at the lower end of the distribution may not be of great concern.
- If we wish to preserve a positive range of values, one can easily introduce a transformation to the data. For example, apply `fitting` not to the physical variable X but rather $Y=\ln(X)$, based on the first four moments of Y. Then the reverse transformation $X=\exp(Y)$ will still be positive.
- The routine `fitting` is intended for applications where the true distribution is not too different than a Weibull model would predict. In such cases we may expect the nonlinear terms (proportional to c_0, c_2, and c_3) in Eq. 4.1 to be relatively small on average relative to the linear term. Thus, compared to the range of likely variation of X values, c_0 may seem to lie rather "close" to zero.

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Appendix A

Driver Source Code Listing

```
C=====================================================================
C
C    THIS PROGRAM DEMONSTRATES THE USE OF THE SUBROUTINES
C
C        CALMOM ...   CALCULATES FOUR MOMENTS OF A GIVEN INPUT DATA SET
C        FITTING...   USES  THESE  MOMENTS  TO  FIT  A  GENERALIZED
C                     (GAUSSIAN, GUMBEL, WEIBULL) DISTRIBUTION FUNCTION
C
C    INPUT FILES:     DRIVER.DAT... INPUT DATA USED TO ESTIMATE MOMENTS
C                     DRIVER.IN... CONTROL DATA USED IN CALLING FITTING
C
C                     (NOTE: SAMPLE EXAMPLE FILES *.DAT AND *.IN SHOULD
C                     BE COPIED TO  DRIVER.DAT  AND  DRIVER.IN   BEFORE
C                     EXECUTING)
C
C    OUTPUT FILE:     DRIVER.OUT
C
C    USAGE:           COMPILE AND LINK DRIVER, CALMOM, AND FITTING
C
C=====================================================================
      PROGRAM DRIVER
C=====================================================================
      IMPLICIT REAL*8 (A-H,O-Z)
      PARAMETER ( NDMAX = 20000 , NXMAX = 2000 )
      DIMENSION DATA(NDMAX),CDF(NXMAX),X(NXMAX),XMOM(4),PMOM(10)
C
C----------------------------------------OPEN FILES FOR INPUT AND OUTPUT
      IODAT  = 10
      IOIN   = 11
      IOOUT  = 12
      OPEN(IODAT,FILE='driver.dat')
```

```
      OPEN( IOIN,FILE='driver.in')
      OPEN(IOOUT,FILE='driver.out')
C
C-------------------------------------------------------READ CONTROL DATA
C
C          XMIN          LOWER THRESHOLD DATA VALUE USED IN THE  ANALYSIS
C          IFLAG         INPUT/OUTPUT FLAG:
C                              IFLAG = 0.....FINDS X FOR INPUT CDF VALUES
C                              IFLAG = 1.....FINDS CDF FOR INPUT X VALUES
C          ITYPE         CONTROL VARIABLE  TO  CHOOSE   THE  GENERALIZED
C                        DISTRIBUTION TYPE
C                      ITYPE = 1 :FIT GENERALIZED GAUSSIAN DISTRIBUTION
C                      ITYPE = 2 :FIT GENERALIZED GUMBEL   DISTRIBUTION
C                      ITYPE = 3 :FIT GENERALIZED WEIBULL  DISTRIBUTION
C          X(NXMAX)      LEVELS OF X AT WHICH DISTRIBUTION IS REPORTED
C          CDF(NXMAX)    CDF VALUES  (NON-EXCEEDENCE  PROBABILITIES)  FOR
C                              EACH X LEVEL
C          NXMAX         MAXIMUM  NUMBER  OF X OR  CDF  VALUES  REQUESTED
C          NX            ACTUAL   NUMBER  OF X OR  CDF  VALUES  REQUESTED
C
      READ(IOIN,*) XMIN,IFLAG,ITYPE
      IF (ITYPE .NE. 3) XMIN = 0.d0
      IF (IFLAG.EQ.0) THEN
           DO 10 IX = 1,NXMAX
   10      READ(IOIN,*,ERR=30,END=30) CDF(IX)
C                                                 READ CDF VALUES IF IFLAG=0
      ELSE
           DO 20 IX = 1,NXMAX
   20      READ(IOIN,*,ERR=30,END=30)   X(IX)
C                                                 READ   X VALUES IF IFLAG=1
      ENDIF
   30      NX = IX - 1

C
C---------------------------------------------------------READ INPUT DATA
C
C          DATA(NDMAX)   ARRAY OF INPUT  DATA FOR WHICH MOMENTS ARE FOUND
C          NDMAX         MAXIMUM NUMBER OF INPUT DATA PERMISSIBLE
C          NDATA         ACTUAL NUMBER OF INPUT DATA
C
      NDATA=0
   40 READ(IODAT,*,ERR=50,END=50) X1
      NDATA=NDATA+1
      DATA(NDATA)=X1
      GO TO 40
```

```
   50 CONTINUE
C
C----------------------------------------CALL CALMOM TO ESTIMATE MOMENTS
C
C          XMOM(4)       ARRAY OF FOUR MOMENTS COMPUTED FROM DATA:
C                            XMOM(1) = MEAN
C                            XMOM(2) = STANDARD DEVIATION
C                            XMOM(3) = SKEWNESS
C                            XMOM(4) = KURTOSIS
C
      CALL CALMOM(XMOM,DATA,NDATA,NDMAX,XMIN,ITYPE)
C
C------------------------------------------------------------WRITE OUTPUT
C
      IF (ITYPE .EQ. 3) THEN
      WRITE(IOOUT,900) '    Lower Threshold Value:', XMIN
      ENDIF
      WRITE(IOOUT, * ) ' Number of Data Processed:', NDATA
      WRITE(IOOUT, * ) '  '
      WRITE(IOOUT, * ) '       **  MOMENT RESULTS  **'
      WRITE(IOOUT, * ) '  '
      WRITE(IOOUT,900) '                        Mean:', XMOM(1)
      WRITE(IOOUT,900) '      Standard Deviation:', XMOM(2)
      WRITE(IOOUT,900) '                Skewness:', XMOM(3)
      WRITE(IOOUT,900) '                Kurtosis:', XMOM(4)
      WRITE(IOOUT, * ) ' '
C
C--------------------CALL FITTING TO ESTIMATE X FOR GIVEN CDF (IFLAG=0)
C                                        OR CDF FOR GIVEN  X  (IFLAG=1)
C
C          ETOL          ERROR TOLERANCE  IN  MATCHING  OBSERVED  MOMENTS
C                          ... HERE WE ACCEPT 0.01 ERROR---USER CAN ALTER
C
C          PMOM(10)      ARRAY OF  PREDICTED ABSOLUTE MOMENTS  FROM MODEL
C                            PMOM(I) = PREDICTED AVERAGE OF X**N, N=1..10
C
      ETOL = .01D0
      CALL FITTING(ITYPE,XMOM,NDATA,XMIN,X,CDF,NX,PMOM,IFLAG,IOOUT,ETOL)
C
C------------------------------------------------------------WRITE OUTPUT
C
      WRITE(IOOUT, * ) '     ** FRACTILE RESULTS (FITTING) **'
      WRITE(IOOUT, * ) ' '
      WRITE(IOOUT, * ) '                      X:           CDF:'
      WRITE(IOOUT, * ) ' '
```

```
            DO 60 IX = 1 , NX
   60       WRITE(IOOUT,901) X(IX),CDF(IX)
      WRITE(IOOUT, * ) ' '
      WRITE(IOOUT, * ) '   ** PREDICTED MOMENTS (FITTING) **'
      WRITE(IOOUT, * ) ' '
      WRITE(IOOUT, * ) '                      N:      E[X**N]:'
      WRITE(IOOUT, * ) ' '
            DO 70 IX = 1 , 10
   70       WRITE(IOOUT,902) REAL(IX),PMOM(IX)
C
  900 FORMAT(A26, F10.3)
  901 FORMAT(16X,2F10.3)
  902 FORMAT(16X, F10.3,E10.3)
C
      STOP
      END
```

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